Kalman Filtering For Spacecraft Trajectories

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ABSTRACT

This poster presentation showcases the use of Kalman Filtering in determining the trajectory of od spacecraft.

There are two methods for approximating non linear filtering.

First is the use of Extended Kalman Filter (EKF) which uses non linear dynamics to propagate the state estimates as well as the output of the dynamics and output maps to propagate the pseudo error covariance.

Second is the Unscented Kalman Filter (UKF) which does not propagate the error covariance using Ricatti Equation rather constructs the covariance by combining a collection of state estimate samples.

The goal is the comparison of the EKF and the UKF for estimating the trajectory of the satellite.

OBJECTIVE

This research aims at finding the better kind of Kalman Filter for the determination of the spacecraft trajectory. It is based on the illustration and comparison of the UKF and the EKF for orbit determination of satellites.

The orbital dynamics have been formulated in terms of six orbital parameters which are non linear and hence a specified time interval and therefore we consider sampled data EKF (SDEKF) and sampled data UKF (SDUKF) for the spacecraft trajectory estimation.

THEORY

Trajectory determination uses range and angle measurements from a constellation of satellites. The tracking and data relay satellite system (TDRSS) uses satellites in geostationary orbits to track satellites in low Earth orbit (LEO). In this paper a constellation of six spacecraft in circular LEO that tracks a satellite in geosynchronous orbit. The number of available measurements varies with time as the observing spacecraft have a much shorter periods than the target satellite.

The paper mainly focuses on three main problems: ability of the observing constellation to acquire target satellite under poor initial information; the ability to track the target satellites orbit when it remains in an equatorial orbit; and ability of the filters to track the target satellite when it changes inclination away from the equatorial plane.

MEASUREMENT MODELS

The position vector for the target with respect to the center of the earth is r which is formulated as:

$$\ddot{\vec{r}} = \frac{-\mu}{r^3}\vec{r} + \vec{w}$$

Where w is perturbing forces and mu is earth's gravitational parameter.
Now introducing velocity vector as:

$$\dot{\vec{r}} = \vec{v},$$
$$\dot{\vec{v}} = \frac{-\mu}{r^3}\vec{r} + \vec{w}$$

Now when we resolve r, v and w with respect to the inertial reference frame:

$$\vec{r}_{\rm I} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \vec{v}_{\rm I} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \quad \vec{w}_{\rm I} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}.$$

These form the basis for the equations of motion for the satellites.

The SDEKF and SDUKF are two step estimators. Thus the before data updates for the state estimate and the error covariance are given by:

$$\hat{X}^{\rm f}(t) \triangleq \begin{bmatrix} \hat{x}^{\rm f} & \hat{y}^{\rm f} & \hat{z}^{\rm f} & \hat{v}_x^{\rm f} & \hat{v}_y^{\rm f} & \hat{v}_z^{\rm f} \end{bmatrix}^{\rm T}$$

$$P_0^{\rm f}(t) \triangleq \mathcal{E}\left[(X(t) - \hat{X}^{\rm f}(t))(X(t) - \hat{X}^{\rm f}(t))^{\rm T}\right]$$

And the after data updates state estimate and error covariance are:

$$\hat{X}^{\rm da}(kh) \triangleq \begin{bmatrix} \hat{x}^{\rm da} & \hat{y}^{\rm da} & \hat{z}^{\rm da} & \hat{v}_x^{\rm da} & \hat{v}_y^{\rm da} & \hat{v}_z^{\rm da} \end{bmatrix}^{\rm T}.$$

$$P_0^{\rm da}(kh) \triangleq \mathcal{E}[(X(kh) - \hat{X}^{\rm da}(kh))(X(kh) - \hat{X}^{\rm da}(kh))^{\rm T}].$$

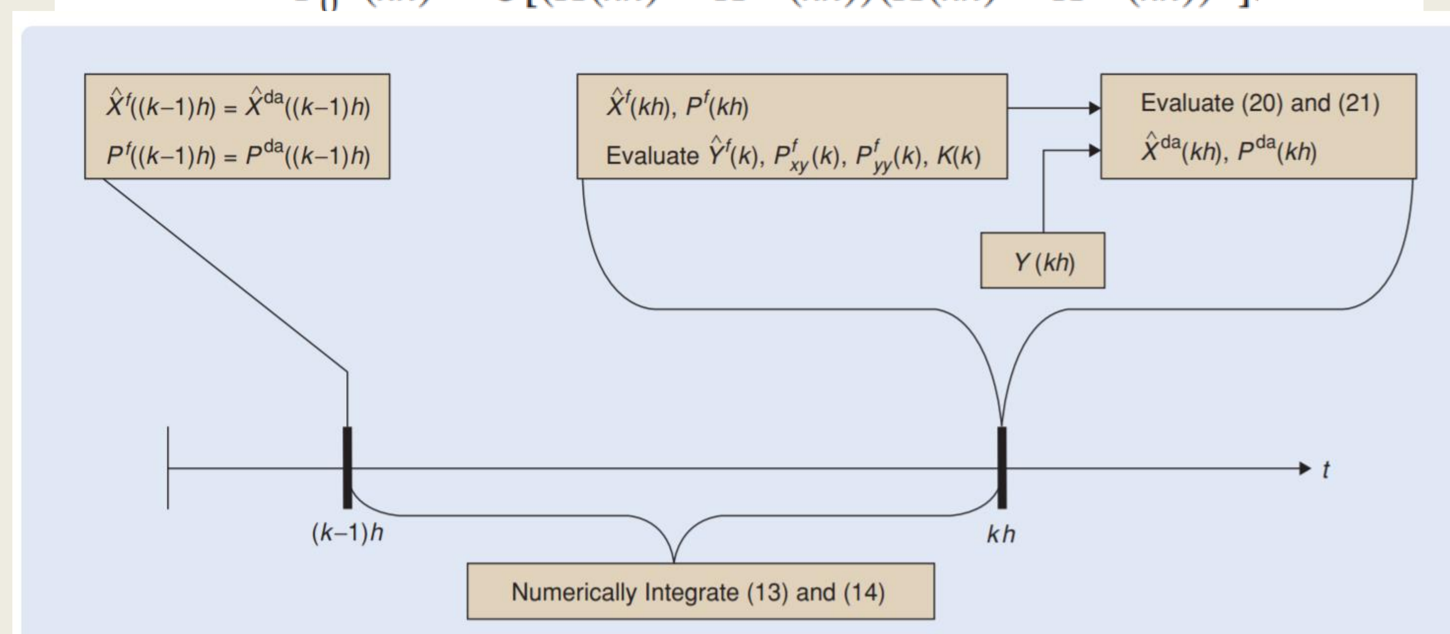

FIGURE 1 Timing diagram for the sampled-data extended Kalman filter. The forecast and data-assimilation steps are assumed to occur in zero time at time $t = kh$.

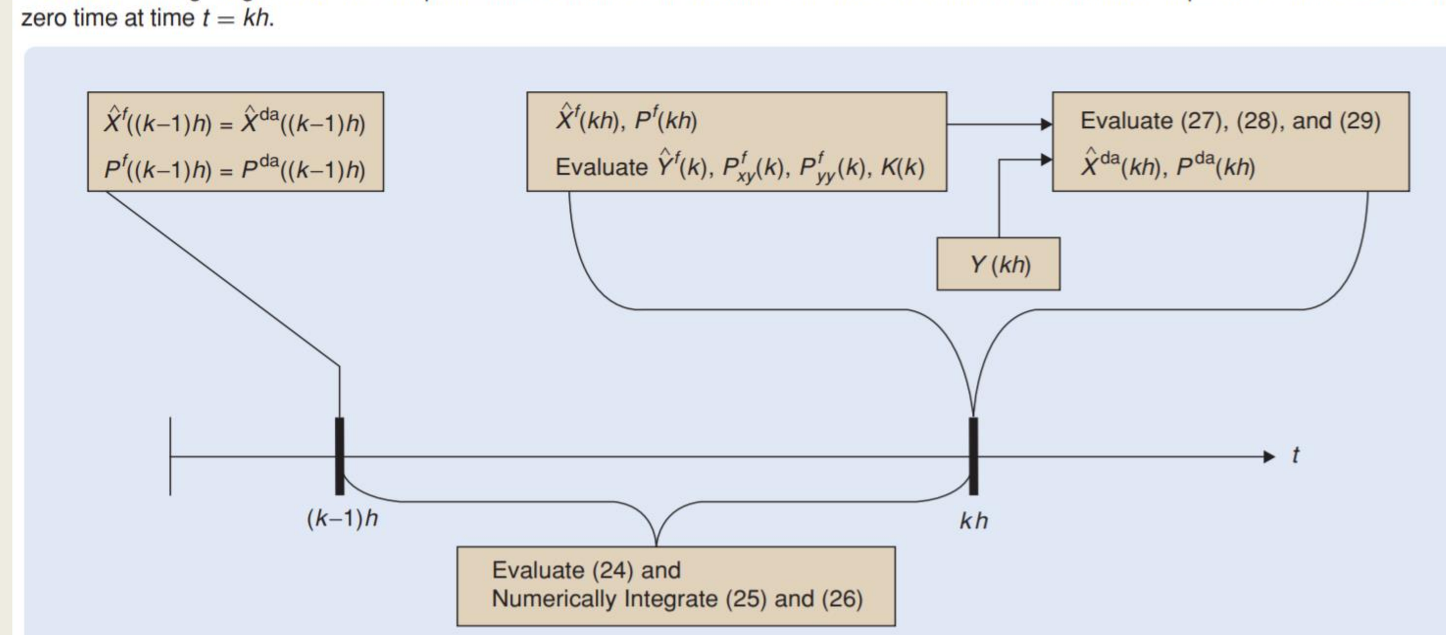

FIGURE 2 Timing diagram for the sampled-data unscented Kalman filter. The forecast and data-assimilation steps are assumed to occur in zero time at time $t = kh$.

RESULTS

The SDEKF and SDUKF provide suboptimal estimates and these are compared using Monte Carlo simulation. The Monte Carlo RMSE simulations are done using:

$$\text{RMSE}_i \triangleq \frac{1}{m}\sum_{j=1}^{m}\left[\sqrt{\frac{1}{k_{\rm f} - k_0 + 1}\sum_{k=k_0}^{k_{\rm f}}\left(X_i(kh) - \hat{X}_{i,j}^{\rm da}(kh)\right)^2}\,\right],$$
$$i = 1, \ldots, 6, \qquad (36)$$

For initialization we set P(0)=diag(100,100,1,1,1,0.1) and used variable step size Runge Kutta Algorithm.

TABLE 1 RMSE$_i$, mean trace (MT), and average CPU processing time for $t \in [500, 1500]$ s and for a 100-run Monte Carlo simulation using the sampled-data extended Kalman filter (SDEKF) and the sampled-data unscented Kalman filter (SDUKF). Range is measured with sample interval $h = 1$ s from six low-Earth-orbit satellites and with Gaussian measurement noise whose standard deviation is 0.1 km. All initial estimates are erroneous by $-90°$. $P^{\rm da}_{3,3}$ and $P^{\rm da}_{6,6}$ are not included in the calculation of MT because their values are much greater than the remaining diagonal entries.

	RMSE$_i$ x (km)	y (km)	z (km)	v_x (km/s)	v_y (km/s)	v_z (km/s)
SDEKF	0.1128	0.2996	63.93	0.0351	0.0841	0.5149
SDUKF	0.5525	0.3175	13.80	0.0958	0.0849	0.2450
	MT (excluding $P^{da}_{3,3}$ and $P^{da}_{6,6}$)					
SDEKF	0.1661					
SDUKF	0.5998					
	Average CPU processing time (ms)					
SDEKF	19.5					
SDUKF	41.8					

Finally, SDUKF fails to track inclination changes but gives more accurate estimates than SDEKF. SDUKF outperforms SDEKF in the z axis; SDEKF outperforms SDUKF in the x axis; and both have similar accuracy for the y axis. SDUKF has almost double the processing time as SDEKF.

DISCUSSION/ CONCLUSION

The paper presents comparison between two filters the SDEKF and SDUKF. For target acquisition SDUKF is more accurate. For SDEKF a diagonal initial covariance is more effective in terms of convergence of the filter. SDUKF tracks the targets eccentricity more accurately than SDEKF.

SDUKF does not detect the changes in the inclination when the target is maneuvering. When angle measurements are available then SDUKF is more accurate. Finally SDUKF has almost double the processing time as SDEKF.

REFERENCES

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